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Total Number of Pages: 02

Course: M.Sc.I
Sub_Code: FMCC602

6th Semester Regular Examination: 2024-25

SUBJECT: COMPLEX ANALYSIS

BRANCH(S): M.Sc.I(MC)

Time: 3 Hours

Max Marks: 70

Q.Code: S144

Answer Question No.1 (Part-I) which is compulsory, any five from rest (Part-II)
The figures in the right hand margin indicate marks.

Part-I

Q1 Answer the following questions :

(2 x 10)

- Find the fixed points for Mobius transformation.
- Determine the real and imaginary part of i^i .
- Obtain the power series representation of $\cos z$.
- Check whether the function $f(z) = |z|^2$ is differentiable at $(0, 0)$.
- State Cauchy integral theorem.
- State fundamental theorem of algebra.
- Define entire function. Give an example of it.
- What do you mean by zeros of analytic functions? Explain it with suitable counterexample.
- State Schwarz's Lemma.
- Determine all poles of the function $f(z) = \frac{\sin z}{\cos z}$.

Part-II

Long Answer Type Questions (Answer Any five)

- Q2** a) Determine the roots of $z^5 = 1$ and plot them. **(5 + 5)**
b) Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = 2$ into the points $w_1 = 1, w_2 = i, w_3 = -1$.
- Q3** a) Find the radius of convergent of $\sum_{n=1}^{\infty} (-3)^n z^{2n}$. **(5 + 5)**
b) Using Argument principle evaluate, $\int_C \frac{z+i}{z^2+2iz-4} dz, C = \{z: |z+1+i| = 2\}$.
- Q4** a) Write Cauchy-Riemann equations in polar coordinates. **(5 + 5)**
b) Write a short note on Maximum modulus principle.
- Q5** a) State and prove Morera's theorem. **(5 + 5)**
b) Evaluate the integral $\oint_C \frac{e^z \sin z}{(z-\frac{\pi}{2})^3} dz, C: |z| = 2$ in counter clockwise sense.

Q6 a) State Liouville's theorem. Prove or disprove that the complex valued function $f(z) = \cos z + z + 1$ is bounded. **(5 + 5)**

b) If γ denotes the positive oriented boundary of square whose sides lies along the line $x = \pm 2$ & $y = \pm 2i$, then evaluate the integral $\oint_{\gamma} \frac{\cos z}{z(z^2+8)} dz$.

Q7 a) Evaluate the integrals $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ using Residue Theorem. **(5 + 5)**

b) Using Rouché's theorem, determine the number of zeros according to its multiplicity of the polynomial $z^{10} - 6z^9 - 3z + 1$.

Q8 a) Develop $f(z) = \frac{2z-3i}{z^2-3iz-2}$ in a Laurent series valid for $1 < |z| < 2$. **(5 + 5)**

b) Using residue theorem solve the real integral $\int_{-\infty}^{\infty} \frac{\cos x}{(x+a)^2+b^2} dx, b > 0$.